Instructions

- Answer **all** questions in the spaces provided.
- Write your responses in English.
- In all questions where a numerical answer is required, you should only round your answer when instructed to do so.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Data analysis

Question 1 (5 marks)

Table 1, below, shows the prices in dollars, *price*, for a sample of 20 homes sold in an inner Melbourne suburb during 2017.

The *type* of home sold is either an apartment or a house.

Table 1

***Price* ($)**	***Type***
350 000	apartment
490 000	apartment
500 000	apartment
620 000	apartment
720 000	apartment
830 000	apartment
875 000	apartment
995 000	apartment
1 100 000	apartment
1 520 000	apartment
800 000	house
840 000	house
920 000	house
920 000	house
1 010 000	house
1 263 000	house
1 398 000	house
1 460 000	house
1 540 000	house
1 540 000	house

Data: Adapted from <www.kaggle.com/datasets/anthonypino/melbourne-housing-market>

a. Find the median, in dollars, of the variable *price*. 1 mark

b. State whether the variable *type* is numerical, nominal or ordinal. 1 mark

c. **i.** Complete the table below by finding the standard deviation, to the nearest whole number, for the sale price of apartments in the sample. 1 mark

Table 2

Type of home	Standard deviation of sale price ($)
house	300 911
apartment	

ii. Using the information in Table 2, comment on the relative spread in the distribution of the sale prices of houses compared with apartments in this sample. 1 mark

d. Table 3, below, shows the percentage of houses and apartments with prices in the given ranges. Some information is missing.

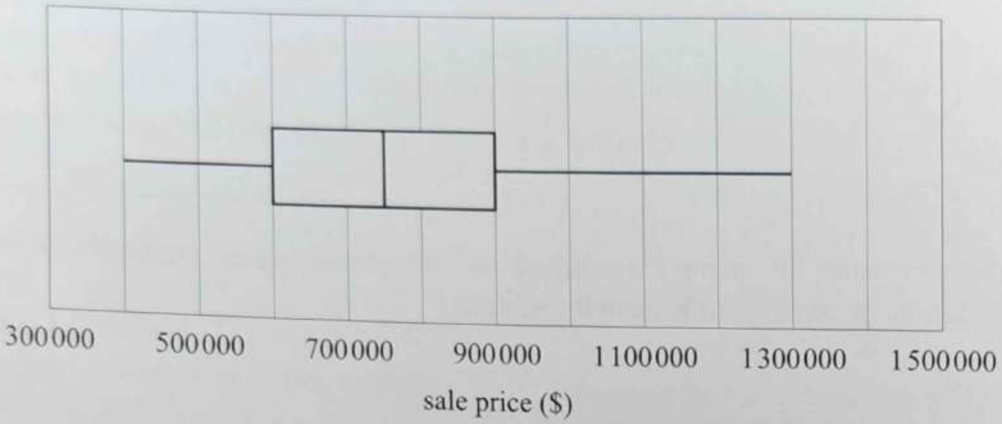
Use the data from Table 1 to complete Table 3. 1 mark

Table 3

Price range ($)	Type of home	
	House (%)	Apartment (%)
less than 600 000		
from 600 000 to 1 000 000	40	
more than 1 000 000		
Total	100	100

Question 2 (2 marks)

A boxplot for the sale prices of a sample of 203 homes is shown.



a. Calculate the range of the sale price data in the boxplot. 1 mark

b. Calculate the upper fence for the sale price data in the boxplot. 1 mark

Question 3 (2 marks)

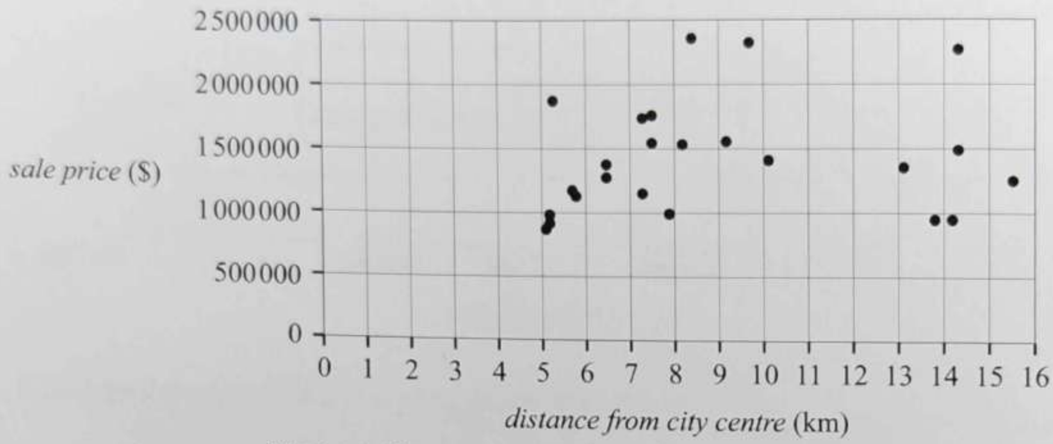
The sale prices for homes in another suburb are normally distributed with a mean of $1 400 000.

A home in this suburb that sold for $952 000 has a standardised score of $z = -1.60$

Using the 68–95–99.7% rule, calculate the percentage of homes sold in this suburb with a sale price between $560 000 and $1 680 000.

Question 4 (8 marks)

The scatterplot below shows the *sale price* of a home, in dollars, against the distance of the home from the city centre of Melbourne, in kilometres, *distance from city centre*. The sample consists of three-bedroom homes sold between 2016 and 2018.



Data: Adapted from <www.kaggle.com/datasets/anthonypino/melbourne-housing-market>

The equation of the least squares line for the data in the scatterplot is

$$\textit{sale price} = 1\,765\,353 - 35\,054 \times \textit{distance from city centre}$$

The coefficient of determination is 0.0806

a. Identify the explanatory variable in the least squares equation. 1 mark

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b. Calculate the value of the correlation coefficient r. Round your answer to three decimal places. 1 mark

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c. Use the equation of the least squares line to predict the *sale price* for a three-bedroom home, located **in the city centre of Melbourne**, sold between 2016 and 2018. 1 mark

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d. Jocelyn wants to sell her three-bedroom home located two kilometres from the city centre of Melbourne.

Would the predicted *sale price* be an example of interpolation or extrapolation? Briefly explain your answer. 1 mark

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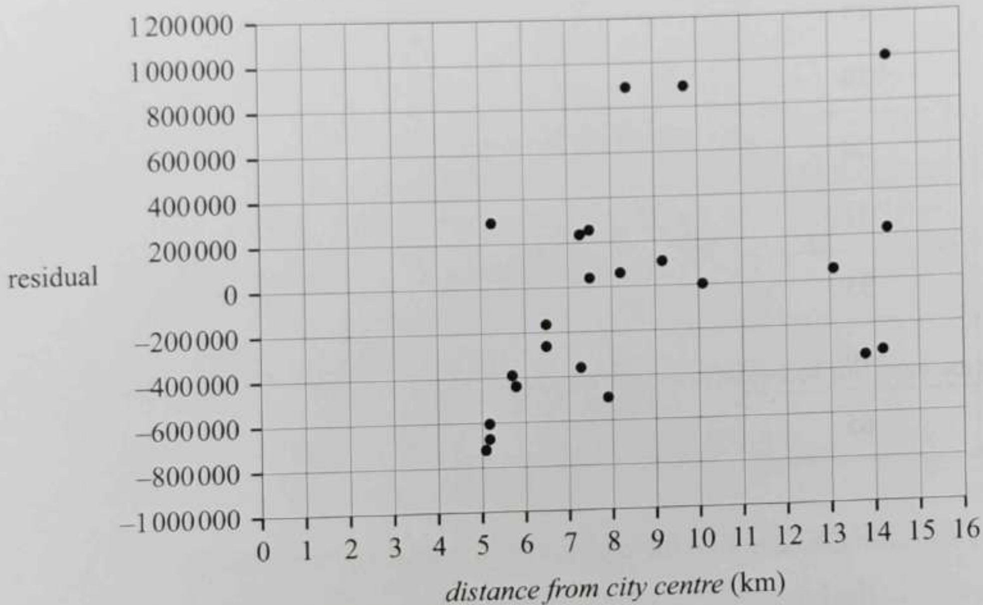
e. Describe the linear association between *sale price* and *distance from city centre* in terms of its strength and direction. Answer in the table below. 2 marks

strength	
direction	

f. A residual plot associated with the least squares line is shown below.

It is missing one point.



The residual associated with the home that is furthest from the city centre of Melbourne is missing from the residual plot. The home is 15.5 km from the city centre and sold for $1 250 000.

i. Show that the value of the missing residual is 27 984. 1 mark

ii. Plot the residual from **part i** by placing an X on the residual plot above. 1 mark

Question 5 (3 marks)

The table below shows *sale price* and the number of days on the market before sale, *days*, for a sample of 10 apartments sold in a particular suburb.

Sale price ($)	*Days*
950 000	15
925 000	18
900 000	23
900 000	24
905 000	26
750 000	28
680 000	31
800 000	35
590 000	46
600 000	65

a. Use the data in the table above to find the equation of the least squares line.

Write your answers in the boxes below, rounding both values to four significant figures. 2 marks

sale price = ☐ + ☐ × *days*

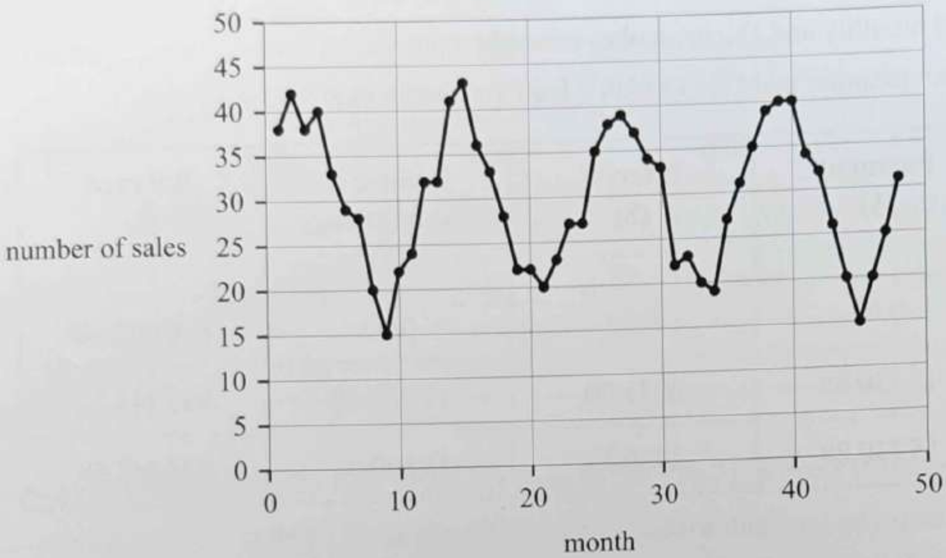
b. For this data, Pearson's correlation coefficient is $r = -0.866$, rounded to three decimal places.

Explain the meaning of the coefficient of determination, as a whole percentage, in the context given in this question. 1 mark

Question 6 (4 marks)

The time series plot below shows the number of homes sold in a town each month over a four-year period.

Month 1 is January 2016 and month 48 is December 2019.



a. Excluding any possible outliers, identify **two** qualitative features of the time series plot. 2 marks

b. The total number of sales in each of the four years is given in the table below.

Year	Total number of sales
2016	361
2017	354
2018	358
2019	357

A seasonal index can be calculated for each month based on the four-year period.

Calculate this seasonal index for September, the ninth month in the calendar year. Round your answer to three decimal places. 2 marks

Recursion and financial modelling

Question 7 (4 marks)

Declan is a filmmaker and content creator.

He has taken out a reducing balance loan to fund a new production.

Interest is calculated monthly and Declan makes monthly repayments.

Three rows of the amortisation table for Declan's loan are shown below.

Payment number	Payment ($)	Interest ($)	Principal reduction ($)	Balance ($)
0	0.00	0.00	0.00	850000.00
1	15730.88	2975.00	12755.88	837244.12
2	15730.88	2930.35	12800.53	824443.59

a. What amount, in dollars, did Declan borrow? 1 mark

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b. Why is the interest associated with payment 2 lower than the interest associated with payment 1? 1 mark

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c. The interest rate on Declan's loan is 4.2% per annum, compounding monthly.

Using the values in the table, complete the table below.

Round all values to the nearest cent. 1 mark

Payment number	Payment ($)	Interest ($)	Principal reduction ($)	Balance ($)
0	0.00	0.00	0.00	850000.00
1	15730.88	2975.00	12755.88	837244.12
2	15730.88	2930.35	12800.53	824443.59
3	15730.88			

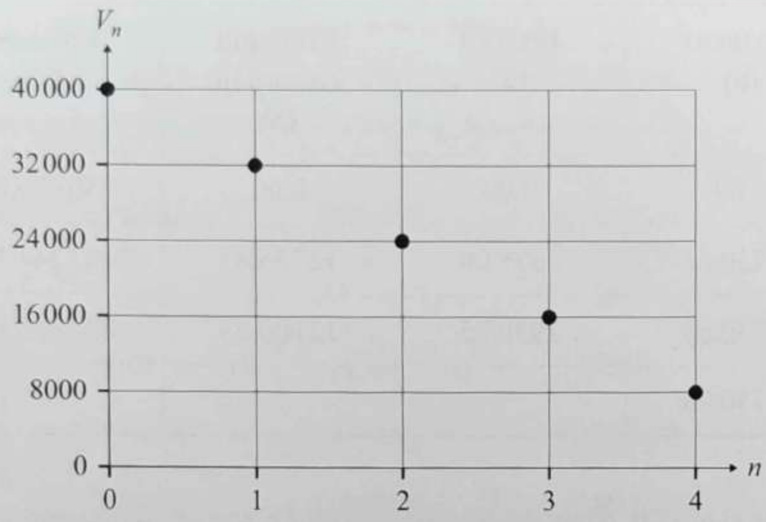
d. The last payment required to fully repay the loan is $15730.71, correct to the nearest cent.

How many payments of $15730.88 did Declan make **before** this final payment? 1 mark

Question 8 (3 marks)

Declan depreciates the value of his lighting equipment using flat rate depreciation.

The graph below shows the value, in dollars, of the lighting equipment, V_n, after n years.



a. The value of the lighting equipment could be modelled by either a recurrence relation or a rule.

i. Write a **recurrence relation** in terms of V_0, V_{n+1} and V_n. 1 mark

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ii. Write a **rule** for V_n in terms of n. 1 mark

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b. What is the annual flat rate depreciation percentage applied to the lighting equipment? 1 mark

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Question 9 (3 marks)

Declan takes out a new loan of \$50 000 to promote his new film.

Interest on this loan compounds weekly and Declan makes weekly repayments of \$75.

a. With these weekly repayments of \$75, suppose the balance of Declan's loan does not change over time.

Determine the weekly interest rate. 1 mark

b. With these weekly repayments of \$75, suppose the balance of Declan's loan now reduces over time.

The balance of the loan, in dollars, after n weeks, L_n, can be determined using a recurrence relation of the form

$$L_0 = 50\,000, \qquad L_{n+1} = RL_n - 75$$

Assume there are exactly 52 weeks in a year.

After one year, Declan owes \$49 565.34

Determine

i. the per annum interest rate, compounding weekly, as a percentage, rounded to two decimal places. 1 mark

ii. the value of R rounded to four decimal places. 1 mark

Question 10 (2 marks)

Using profits from his recent film, Declan invests \$650 000 into a 10-year annuity.

The annuity earns interest at 6.4% per annum, compounding quarterly.

Declan receives a regular quarterly payment from the annuity.

Halfway through the 10-year annuity, Declan writes a recurrence relation to represent the quarter-to-quarter balance for the remainder of the annuity.

Let D_n be the balance of Declan's annuity n quarters after the **halfway** point of the annuity.

Complete the recurrence relation below in terms of D_0, D_{n+1} and D_n that can model this balance.

$D_0 =$ [ ], $D_{n+1} = 1.016 \times D_n +$ [ ]

(Answer in the boxes above.)

Matrices

Question 11 (3 marks)

An early learning centre contains three rooms, Nursery (N), Toddler (T) and Pre-kinder (P).

The Nursery and Toddler rooms each have capacity for eight children and the Pre-kinder room has capacity for 20 children, as shown in matrix C below.

$$C = \begin{bmatrix} 8 \\ 8 \\ 20 \end{bmatrix} \begin{matrix} N \\ T \\ P \end{matrix}$$

Matrix E shows enrolment numbers for each room for one week, Monday to Friday.

$$\begin{matrix} & Mon & Tue & Wed & Thu & Fri \end{matrix}$$
$$E = \begin{bmatrix} 6 & 8 & 8 & 8 & 5 \\ 7 & 8 & 7 & 8 & 6 \\ 18 & 18 & 17 & 15 & 13 \end{bmatrix} \begin{matrix} N \\ T \\ P \end{matrix}$$

a. State the order of matrix E.

Write your answer in the boxes provided. 1 mark

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b. The following matrix multiplication has been completed to determine a new matrix, W.

$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \times E = W$$

What information does matrix W provide regarding enrolments at the early learning centre? 1 mark

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c. It has been decided that the capacity of the Nursery room will be increased by 25% and the capacity of the Toddler room will be increased by 50%. The capacity of the Pre-kinder room will be reduced by 10%.

The new capacities for the three rooms (C_{new}) can be determined from the matrix product

$$C_{new} = FC$$

where F is a diagonal matrix.

Write down the matrix F. 1 mark

$F =$

Question 12 (2 marks)

The early learning centre contains three rooms, Nursery (N), Toddler (T) and Pre-kinder (P).

From one year to the next, children can move between rooms, stay in the same room, or may leave (L) the centre. The following transition matrix, M, shows the expected proportion of children who will move between categories or stay in the same category from one year to the next.

$$M = \begin{array}{c} \textit{this year} \\ \begin{array}{cccc} N & T & P & L \end{array} \\ \begin{bmatrix} 0.25 & 0 & 0 & 0 \\ 0.625 & 0.25 & 0 & 0 \\ 0 & 0.625 & 0.1 & 0 \\ 0.125 & 0.125 & 0.9 & 1 \end{bmatrix} \end{array} \begin{array}{l} N \\ T \\ P \\ L \end{array} \quad \textit{next year}$$

a. The number of children expected to be in each of the four categories, from one year to the next, can be calculated using the matrix recurrence relation

$$S_{n+1} = MS_n$$

where S_n represents the expected number of children in each of the four categories at the start of year n.

The state matrix S_{2024}, shown below, gives the number of children in each category at the start of 2024.

$$S_{2024} = \begin{bmatrix} 4 \\ 15 \\ 15 \\ 27 \end{bmatrix} \begin{array}{l} N \\ T \\ P \\ L \end{array}$$

Find S_{2023}. 1 mark

b. From the start of 2025, new children commenced in the early learning centre at the start of each year.

A new matrix recurrence relation for determining the expected number of children in each of the four categories from one year to the next is

$$S_{n+1} = MS_n + B$$

where

$$B = \begin{bmatrix} 12 \\ 5 \\ 10 \\ 0 \end{bmatrix} \begin{matrix} N \\ T \\ P \\ L \end{matrix}$$

gives the number of new children enrolled in each room of the early learning centre at the start of each year.

Given the state matrix

$$S_{2024} = \begin{bmatrix} 4 \\ 15 \\ 15 \\ 27 \end{bmatrix} \begin{matrix} N \\ T \\ P \\ L \end{matrix}$$

find the expected total number of children to be enrolled in the early learning centre at the start of 2026. Round your answer to the nearest whole number. 1 mark

Question 13 (4 marks)

The early learning centre also offers a 10-week activity program for four-year-old children. There are 27 children enrolled in the program. They participate in three different activities over the 10 weeks. The activities are cooking (C), gardening (G) and music (M).

The transition matrix K, shown below, gives the expected proportion of children in the program who will change activities from one week to the next.

$$\begin{array}{c} \textit{this week} \\ \begin{array}{ccc} C & G & M \end{array} \\ K = \begin{bmatrix} 0 & 0.76 & 0.36 \\ 0.55 & 0 & 0.64 \\ 0.45 & 0.24 & 0 \end{bmatrix} \begin{array}{l} C \\ G \\ M \end{array} \textit{ next week} \end{array}$$

a. What do the values on the leading diagonal in matrix K indicate? 1 mark

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b. In Week 1 of the program, all 27 children participate in cooking (C).

i. Calculate the expected percentage of children who will participate in cooking in Week 10 of the program. Round your answer to one decimal place. 1 mark

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ii. Find the expected number of children who will participate in gardening (G) in Week 3 of the program and then move across to music (M) in Week 4 of the program. Round your answer to the nearest whole number. 2 marks

Question 14 (3 marks)

The early learning centre runs seven different activities during its 40-day holiday program.

The activities are cooking (C), drama (D), gardening (G), lunch (L), music (M), reading (R) and sport (S).

The timetabled order of the activities for day one of the holiday program is shown below.

9 am	10 am	11 am	12 pm	1 pm	2 pm	3 pm
C	D	G	L	M	R	S

The timetabled order of the activities for day one is also shown in matrix X below.

$$X = \begin{bmatrix} C \\ D \\ G \\ L \\ M \\ R \\ S \end{bmatrix}$$

Matrix P, shown below, is a permutation matrix used to determine the timetabled order of activities from one day to the next.

$$P = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

A column matrix containing the timetabled order of activities on one day is multiplied by matrix P to determine the timetabled order of activities for the next day.

a. State the activities that are always held at the same time on each day of the program. 1 mark

b. Determine the timetabled order of the seven activities on day three of the program. 1 mark

c. P^4 is an identity matrix.

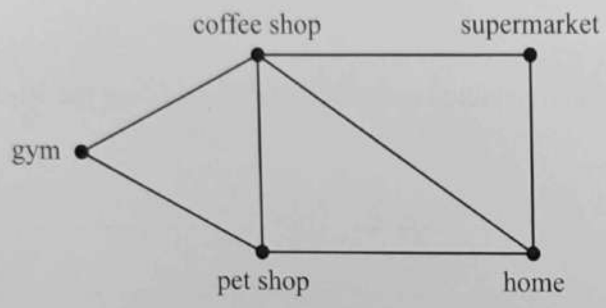
Explain what this means for the timetabled order of the activities over the 40-day holiday program. 1 mark

Networks and decision mathematics

Question 15 (4 marks)

Frances lives in a housing estate.

On the graph below the vertices represent her favourite locations, and the edges represent the roads between them.



a. Calculate the sum of the degrees of all the vertices in this graph. 1 mark

b. Euler's formula, $v + f = e + 2$, holds for this graph.

Complete the formula by writing the appropriate numbers in the boxes below. 1 mark

$\boxed{} + \boxed{} = \boxed{} + 2$

v f e

c. Frances is at the gym. She would like to visit each of the other locations once and end at her home.

What is the mathematical term used to describe this route? 1 mark

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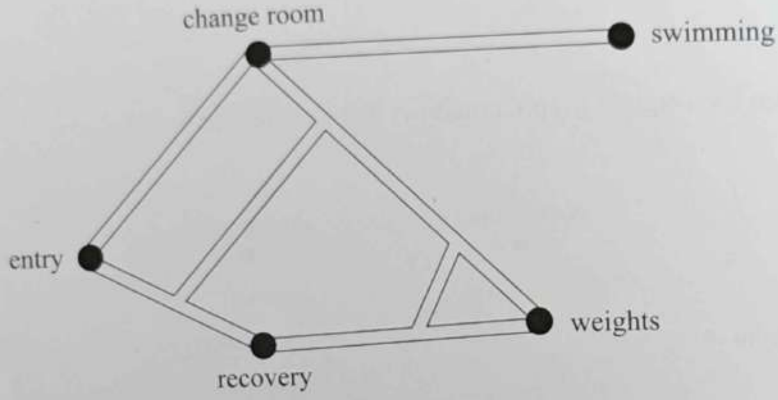
d. Using edges from the original graph, construct a spanning tree below. 1 mark

Question 16 (2 marks)

The map below shows the passages connecting five areas in the gym: entry (E), recovery (R), weights (W), change room (C) and swimming (S).



A network can be constructed from this map.

An adjacency matrix for this network is shown below.

Some values in the matrix are given as x, y and z.

$$\begin{array}{c} \\ E \\ R \\ W \\ C \\ S \end{array}\begin{array}{c} \begin{array}{ccccc} E & R & W & C & S \end{array} \\ \begin{bmatrix} 0 & x & 2 & 2 & 0 \\ x & 1 & y & 2 & 0 \\ 2 & y & 1 & z & 0 \\ 2 & 2 & z & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

In this matrix, the '2' in row E, column C indicates that there are two ways of moving from the entry to the change room without passing through another area or backtracking.

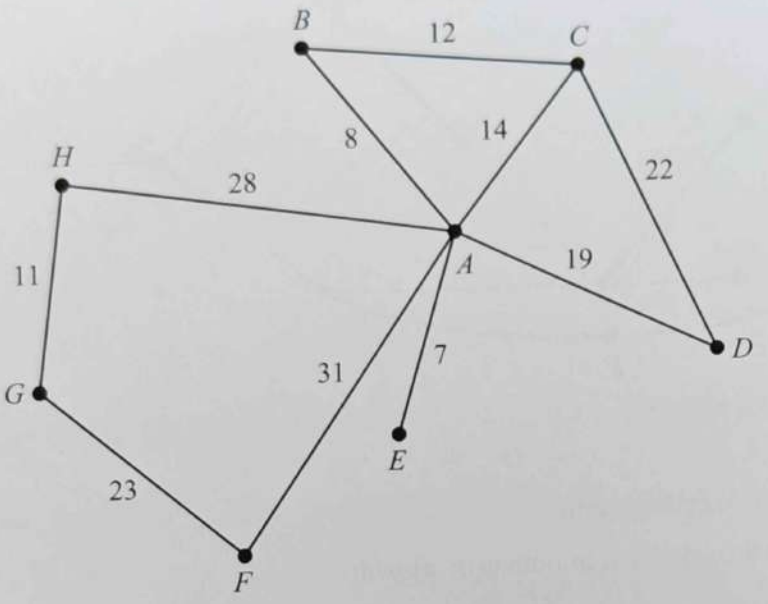
Write the values of x, y and z in the boxes below.

$x =$ ☐ $y =$ ☐ $z =$ ☐

Question 17 (2 marks)

The gym is hosting a competition in which competitors will complete activities at eight different stations.

On the network below, the vertices represent the stations. The edges represent the walkways between the stations and the numbers show the distance, in metres, between them.



The gym owner would like to make sure that all the walkways are clear before the competition starts.

The gym owner would like to begin and end the inspection of the walkways at station A.

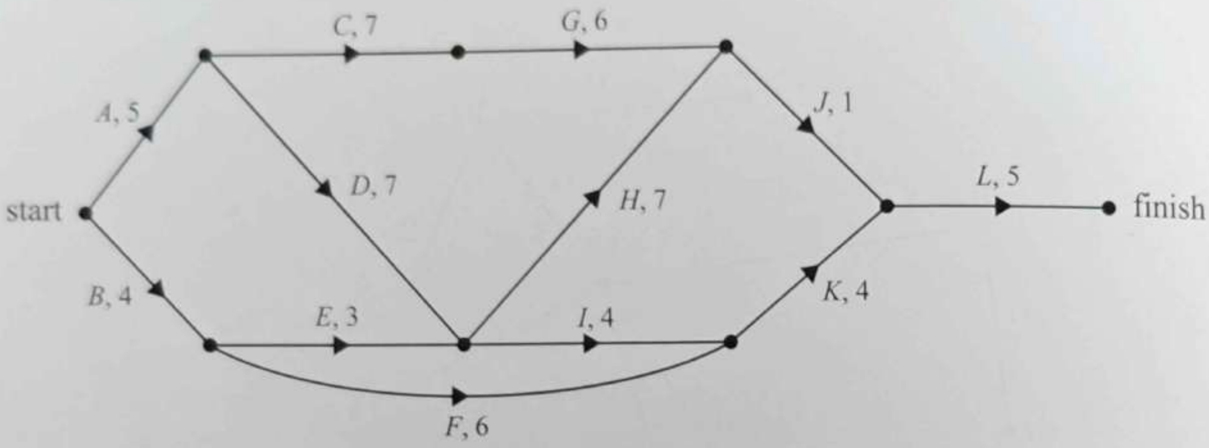
a. Explain, with reference to the degrees of the vertices, why the gym owner's intended route must involve some repeated edges. 1 mark

b. What is the minimum distance, in metres, that the gym owner will cover when completing the inspection? 1 mark

Question 18 (4 marks)

Frances is constructing a home gym. This project requires 12 activities, A to L, to be completed.

The activity network below shows each activity and its completion time in days.



a. This network contains two critical paths.

State the activities that are common to both critical paths. 1 mark

b. Determine the latest start time, in days, for activity E. 1 mark

c. Which activity has the longest float time? 1 mark

d. The table below shows five activities that can have their completion time reduced. It shows the maximum reduction time (days) and the additional cost per day, for each of the five activities.

Activity	**Maximum reduction time (days)**	**Additional cost per day ($)**
A	2	500
F	4	150
G	4	150
H	2	300
K	1	100

Frances would like to construct the home gym in three days less than was previously possible.

What is the minimum additional amount Frances will need to pay?

1 mark

End of Question and Answer Book