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Write your **student number** in the boxes above. **Letter**

Mathematical Methods Examination 1

Question and Answer Book

VCE Examination – Wednesday 5 November 2025

- Reading time is **15 minutes**: 9.00 am to 9.15 am
- Writing time is **1 hour**: 9.15 am to 10.15 am

Materials supplied

- Question and Answer Book of 16 pages
- Formula Sheet

Instructions

Students are **not** permitted to bring any technology (calculators or software), or notes of any kind, into the examination room.

Students are **not** permitted to bring mobile phones and/or any unauthorised electronic devices into the examination room.

Contents

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Instructions

- Answer **all** questions in the spaces provided.
- Write your responses in English.
- In all questions where a numerical answer is required, an **exact** value must be given unless otherwise specified.
- In questions where more than one mark is available, appropriate working **must** be shown.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (3 marks)

a. Let $y = x^2\cos(x)$.

Find $\frac{dy}{dx}$. 1 mark

b. Let $f(x) = 6\sqrt{x+1}+5$.

Find the gradient of the tangent to $y = f(x)$ at $x = 8$. 2 marks

Question 2 (2 marks)

Let $g(x)$ be a function defined for $x > -\frac{3}{2}$ so that $g'(x) = \frac{1}{2x+3}$ and $g(1) = 0$.

Find $g(x)$.

Question 3 (6 marks)

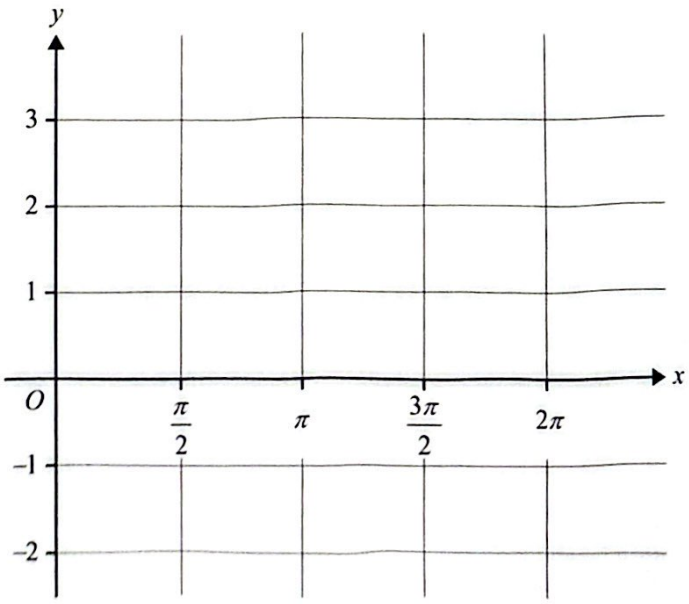
Let $f: [0, 2\pi] \to R$, $f(x) = 2\cos(2x) + 1$.

a. State the range of f. 1 mark

b. Solve $f(x) = 0$ for x. 3 marks

c. Sketch the graph of $y = f(x)$ for $x \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ on the axes below.

Label the endpoints with their coordinates. 2 marks



Question 4 (4 marks)

The probability distribution for the discrete random variable X is given in the table below, where k is a positive real number.

x	0	1	2	3
$\Pr(X=x)$	$\frac{4}{k}$	$\frac{2k}{75}$	$\frac{k}{75}$	$\frac{2}{k}$

a. Show that $k = 10$ or $k = 15$. 2 marks

b. Let $k = 15$.

i. Find $\Pr(X > 1)$. 1 mark

ii. Find $\mathrm{E}(X)$. 1 mark

Question 5 (4 marks)

a. Solve $e^{2x} - 8e^x + 7 = 0$ for x. 2 marks

b. Let $g(x) = e^{2x} - 8e^x + 7$, where $x \in R$.

The function $g(x)$ has exactly one stationary point, a local minimum.

Find the largest value of a such that when g is restricted to the domain $(-\infty, a]$ it has an inverse function. 2 marks

Question 6 (3 marks)

Consider the binomial random variable $X \sim \text{Bi}\left(6, \frac{1}{4}\right)$.

a. Find $\text{var}(X)$. 1 mark

b. Determine $\Pr(X \geq 5)$.
Give your answer in the form $\frac{a}{2^b}$, where $a, b \in Z$. 2 marks

Question 7 (6 marks)

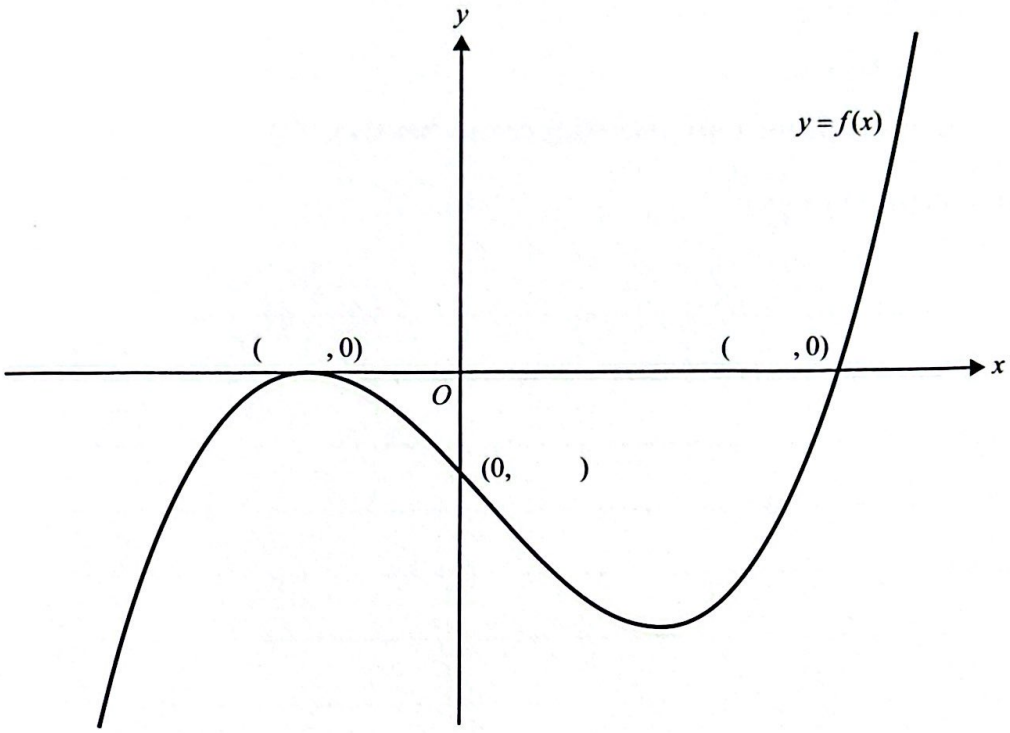
Let $f: R \to R, f(x) = x^3 - x^2 - 16x - 20$.

a. Verify that $x = 5$ is a solution of $f(x) = 0$. 1 mark

b. Express $f(x)$ in the form $(x+d)^2(x-5)$, where $d \in R$. 2 marks

c. Consider the graph of $y = f(x)$, as shown below.

Complete the coordinate pairs of all axial intercepts of $y = f(x)$. 1 mark



d. Let $g : R \to R,\ g(x) = x + 2$.

i. State the coordinates of the stationary point of inflection for the graph of $y = f(x)g(x)$. 1 mark

ii. Write down the values of x for which $f(x)g(x) \geq 0$. 1 mark

Question 8 (5 marks)

Consider

$$f(x)=\begin{cases}\frac{3}{8}(4-3x) & 0\le x\le\frac{4}{3}\\ 0 & \text{otherwise}\end{cases}$$

a. The continuous random variable X has probability density function $f(x)$.

Find k such that $\Pr(X>k)=\frac{9}{16}$. 3 marks

b. The function $h(x)$ is a transformation of $f(x)$ such that

$$h(x)=mf(x)+n$$

where m and n are real numbers.

Find $\int_0^{\frac{4}{3}} h(x)\,dx$ in terms of m and n. 2 marks

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Examination continues on the next page.

Question 9 (7 marks)

Consider the functions

$$f: R \setminus \{1\} \to R, f(x) = \frac{w^2}{(x-1)^2}$$

and

$$g: R \to R, g(x) = (x-w)^2$$

where $w \in R$.

a. If $w = -3$, find the four solutions to $f(x) = g(x)$. 3 marks

b. Consider the case where $w > 0$.

i. Find, in terms of w, the coordinates of the minimum point of the graph of $y = (x-1)(x-w)$. 2 marks

ii. Hence, or otherwise, find the positive values of w for which $f(x) = g(x)$ has exactly three solutions. 2 marks

End of examination questions

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